International Journal of Theoretical Physics, Vol. 38, No. 10, 1999

Fourier Analysis for Proton–Proton Interaction at High Energy

T. I. Haweel[1] and M. Y. El-Bakry[1]

Received March 23, 1999

The charged-particle multiplicity distribution and the inelastic cross sections for proton–proton (p–p) interactions are studied. The parton two-fireball model based on an impact parameter is adopted. The overlapping function, known to be complicated and nonlinear, is represented using a Fourier series. Cross sections and charged-particle multiplicity distributions are derived. Good agreement is found between theoretical calculations and experimental data at different energies.

1. INTRODUCTION

Much of what we know about the dynamics of multiparticle production comes from the study of multiplicity distributions [1]. Various methods have been suggested by different authors for describing the multiplicity distribution of the particles produced in high-energy interactions at ultrarelativistic energies. Among these are multiplicity scaling [2, 3], the negative binomial distribution (NBD) [4], the two-source model [5], the partially coherent laser distribution (PCLD) [6], Monte Carlo studies of pion distributions from heavy ion collisions [7], the statistical bootstrap model [8], the three-fireball model [9], quark models [10], fragmentation models [11, 12], and many others. In this connection, the multiplicity distribution of proton–proton (p–p) interactions for various energies can be described by a parton two-fireball model as in refs. 13–15, where hadrons are composed of quarks and gluons that collectively can be considered as pointlike particles called partons [16]. This nucleon structure has been used in different mathematical models [17, 18] along with other assumptions to describe hadron–hadron interactions. The partons behave as free pointlike particles in high–energy collisions. Thus

[1] Salalah Teachers Training College, P.O. Box 1905, Postal Code 211, Salalah, Sultanate of Oman; e-mail: tarekih@hotmail.com.

0020-7748/99/1000-260 5$16.00/0

one can use the impulse approximation and deal with cross sections for individual partons. It is assumed that the partons are homogeneously distributed in the nucleon volume, which has a spherical shape at rest.

In this work, a simple yet efficient mathematical model is utilized to study the multiparticle production process in the p–p interaction. It is based on representing the overlapping function (z function), known to be complicated and nonlinear, using a trigonometric Fourier series. The resulting trigonometric function is simple and analytic, and thus facilitates the analysis. The representation can be made arbitrarily accurate by increasing the order of the Fourier series. The resulting multiplicity distributions agree with experimental data.

The rest of the paper is organized as follows: Section 2 reviews the essential features for the parton two-fireball model and introduces the proposed Fourier series approximation. Section 3 reviews the p–p interaction cross sections and demonstrates the efficiency of the proposed analysis in fitting experimental data. The resulting charged-particle multiplicity distribution is considered in Section 4 for different energies. Section 5 concludes the work.

2. MODELING THE p–p INTERACTION

2.1. A Parton Two-Fireball Model:

The impact parameter analysis considered in this paper to study the p–p interaction at high energies employs the parton two-fireball model. During a collision, the majority of parton momentum is carried by the longitudinal component. The nucleons are semitransparent objects and the two interacting nucleons penetrate each other in the incident direction. In p–p collision, only those partons in the overlapping volume of the two protons have a probability to interact. The interaction is stopped in the center-of-mass system (CMS), and therefore the kinetic energy (KE) is consumed in the excitation of the two fireballs produced. Each fireball decays into a number of newly created particles with an isotropic angular distribution in its own rest frame.

2.2. The Overlapping Function

The number of partons participating in the p–p interaction is defined according to an incident impact parameter, particularly at high energies where the Coulomb interaction between the two protons is negligible. The geometrical cross section for p–p interaction is given by the area of a circle of radius $2r$, where it is assumed that the proton at rest is a sphere of radius r. Therefore, the interaction statistical probability for an impact parameter b within an interval db is given by

$$p(b)\,db = \frac{b\,db}{2r^2}, \qquad 0 \le b \le 2r \tag{1}$$

Let us use a dimensionless impact parameter x defined as $x = b/2r$. Therefore, (1) can be written as

$$p(x)\,dx = 2x\,dx, \qquad 0 \le x \le 1 \tag{2}$$

Now we employ the overlapping volume $V(b)$ of ref. 19 (spherical cap) and assume, for the sake of the calculations, that the overlapping volume is given approximately as a clean cut. Then the fraction of partons z participating in the interaction may be written as

$$z(x) = \left(\frac{V(x)}{V_0}\right) = \left[1 - \frac{3}{4}x - \frac{3}{2}x^2 + \frac{5}{4}x^3\right] \tag{3}$$

where V_0 is the volume of the nucleon.

According to (2) and (3), the overlapping function (z-function) distribution may be written as

$$p(z)\,dz = \frac{2z\,dz}{-2.438 - 0.75x^{-1} + 7.125x + 0.75x^2 + 9.375x^3 + 4.6688x^4}, \qquad 0 \le z \le 1 \tag{4}$$

Clearly, (4) shows that the z-function is complicated and nonlinear. Simplifying this function has been the goal of a number of researchers [e.g., 13, 14]. The next section proposes a powerful, simple, and efficient simplification.

2.3. Proposed Trigonometric Fourier Series Representation for the Overlapping Function

As seen from the previous section, the overlapping function is not easy to manipulate mathematically. However, signal processing techniques make it possible to facilitate manipulating such functions. It is shown that the overlapping function is defined over the interval [0, 1]. Such a function may be represented as a trigonometric Fourier series [20] as follows:

$$p(z) = a_0 + \sum_{k=1}^{K} a_k \cos(2\pi kz) + \sum_{k=1}^{K} b_k \sin(2\pi kz) \tag{5}$$

where

$$a_0 = \int_0^1 p(z)\,dz; \qquad a_k = 2\int_0^1 p(z)\cos(2\pi kz)\,dz; \tag{6}$$

$$b_k = 2\int_0^1 p(z)\sin(2\pi kz)\,dz$$

K is the order of the Fourier series representation. Intuitively, as K increases, the representation becomes more accurate. In the limiting case when k tends to infinity, the representation is exact. Trapezoidal numerical integration has been used to evaluate the Fourier series coefficients in (6). The representation may be put in the more compact form [20]

$$p(z) = \sum_{k=0}^{K} c_k \cos(2\pi kz + \theta_k); \qquad c_k = \sqrt{a_k^2 + b_k^2}; \qquad \theta_k = -\tan^{-1}\frac{b_k}{a_k} \tag{7}$$

Several advantages are immediately apparent employing this representation. The accuracy is tunable through the order k. The resulting trigonometric functions are analytic and can be easily differentiated and integrated. The analysis has been facilitated, and, as will be shown in the next sections, the resulting theoretical behavior fits experimental data.

3. p–p INTERACTION CROSS SECTIONS

The p–p interaction cross sections are divided into elastic, σ_{el}, and inelastic, σ_i, with their sum being the total cross section, σ_t. To calculate these cross sections, we must know the limits of each one. The elastic

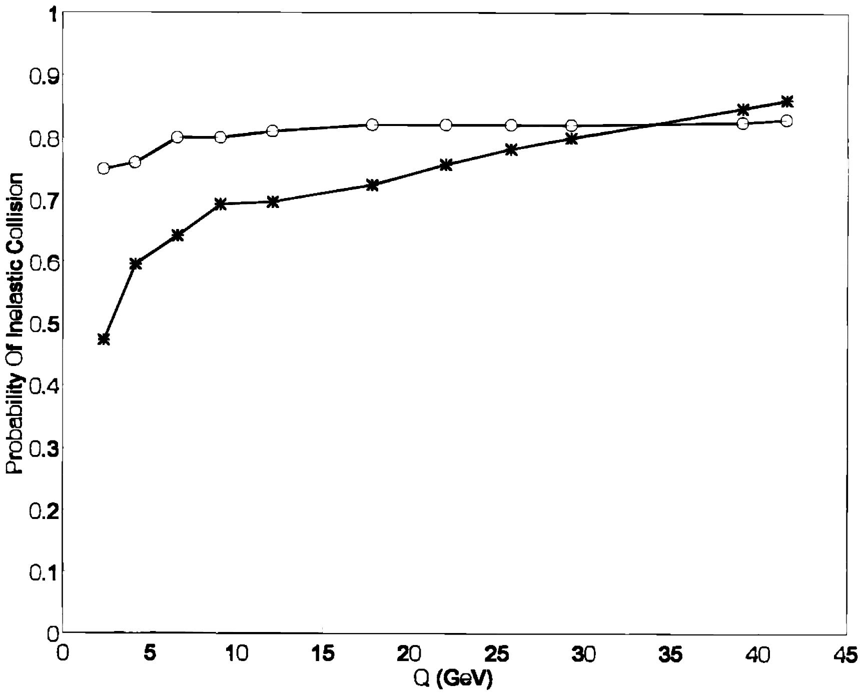


Fig. 1. Probability of inelastic collision. (O) Experimental, (*) theoretical.

collisions are those interactions which create no new particles; in other words, the momentum transferred is not enough for the creation of at least one pion. The inelastic collisions are those interactions in which there is a probability for creation of new particles with at least one pion from each fireball.

The slight increase of the inelastic cross section σ_i with energy observed experimentally [21] can be explained roughly by introducing a threshold value Z_{th} for the elastic collisions. We therefore assume a threshold for elastic scattering corresponding to the exchange of one pion of mass m_π; then the excitation energy in the CMS for each fireball will be

$$M_f - m = \frac{m_\pi}{2} = T_0 Z_{th} \tag{8}$$

where M_f is the fireball rest mass, m is the proton mass at rest, T_0 is the KE of the proton in the CMS before collision, and

$$Z_{th} = \frac{m_\pi}{2T_0} = \frac{m_\pi}{Q} \tag{9}$$

where Q is the total KE in the CMS ($= 2T_0$).

If we assume that the minimum fireball mass required for the creation process (M_{fmin}) is that corresponding to the production of one pion, i.e., with an excitation energy equals to the energy-required for the creation of one pion, say ε, then we have

$$M_{\mathrm{fmin}} = \varepsilon + m = T_0 z_{\min} + m \tag{10}$$

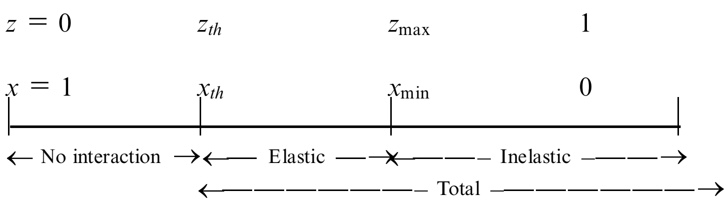
where $z_{\min}$ is the corresponding overlapping function. Hence, elastic and inelastic processes may be classified as

$$M_f < \varepsilon + m \quad \text{(elastic)}; \qquad M_f \geq \varepsilon + m \quad \text{(inelastic)}$$

The value of $z_{\min}$ is

$$z_{\min} = \frac{\varepsilon}{T_0} = \frac{2\varepsilon}{Q} \tag{11}$$

Moreover, if z_{th} corresponds to an impact parameter $x = x_{th}$ and $z_{\max}$ corresponds to $x_{\min}$, then we have



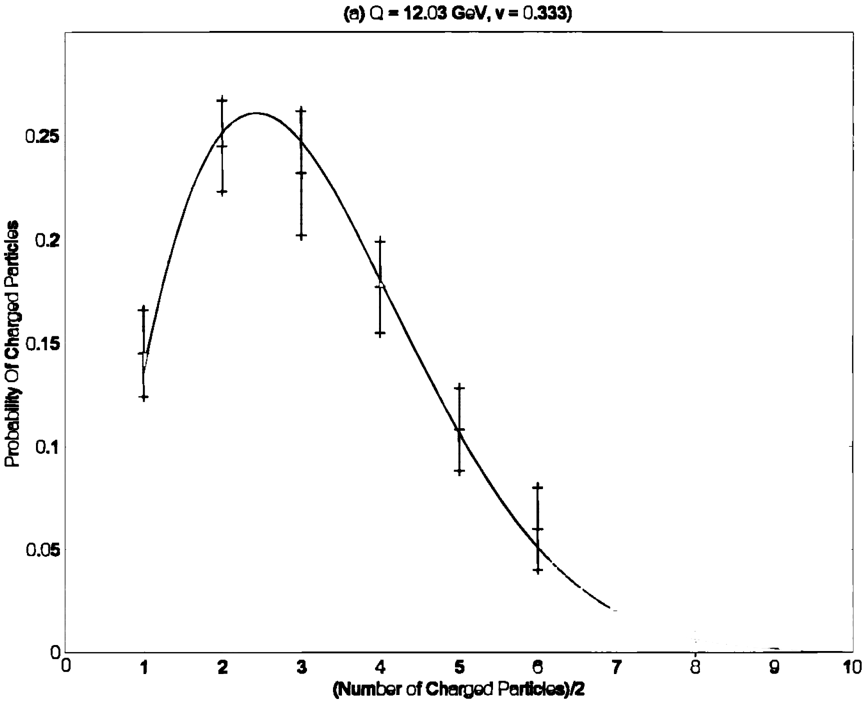


Fig. 2. Charged multiplicity distributions. (⊥) Experimental; (—) theoretical.

Therefore, the probability of the inelastic collisions is given by

$$P_{\text{incl}} = \frac{\sigma_i}{\sigma_t} = \frac{\int_{z_{\min}}^{l} p(z)\, dz}{\int_{z_{th}}^{l} p(z)\, dz} \tag{12}$$

With $p(z)$ as in (7), we have

$$\frac{\sigma_i}{\sigma_t} = \frac{a_0(1 - z_{\min}) + \sum_{k=1}^{K} \frac{c_k}{2\pi k}[\sin(2\pi k + \theta_k) - \sin(2\pi k z_{\min} + \theta_k)]}{a_0(1 - z_{\text{th}}) + \sum_{k=1}^{K} \frac{c_k}{2\pi k}[\sin(2\pi k + \theta_k) - \sin(2\pi k z_{th} + \theta_k]} \tag{13}$$

The probability of elastic collisions is

$$P_{ee} = \frac{\sigma_{el}}{\sigma_t} = 1 - \frac{\sigma_i}{\sigma_t} \tag{14}$$

We have calculated σ_i/σ_t as a function of the free energy Q using $\varepsilon = 0.47$

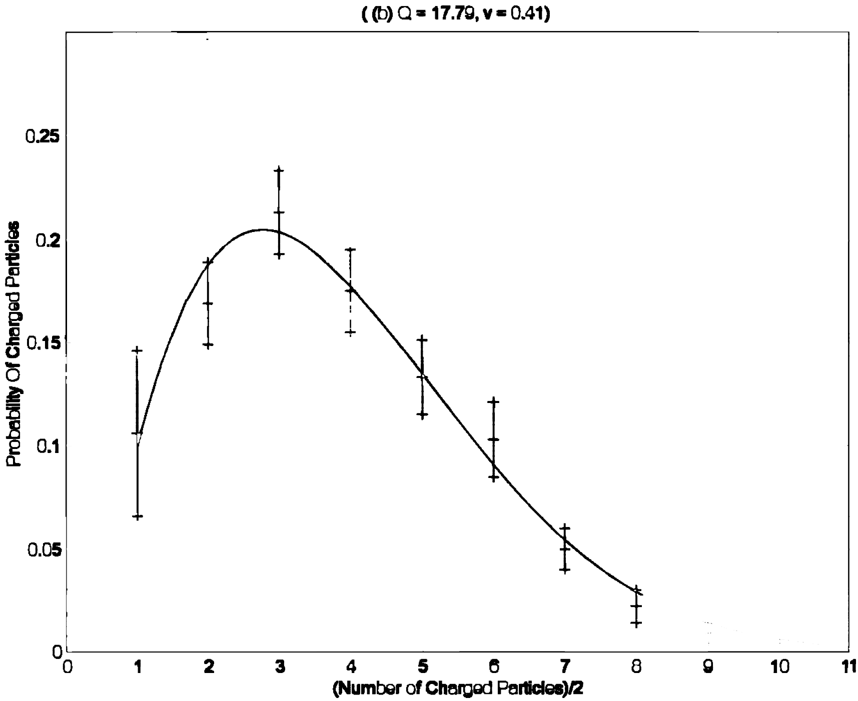


Fig. 2. Continued.

GeV. The results of these calculations, illustrated in Fig. 1, show a good fit with experimental data at high energies, while the fit is not so good at low energies. This means that the inelastic threshold introduced above has a small effect on the cross-section ratios. The disagreement with observations at low energies could be due to the assumptions of the present model, which seem to be valid at high energies only.

4. CHARGED-PARTICLE MULTIPLICITY DISTRIBUTIONS

After collision, the number of pions from each fireball n_s will be given by

$$n_s = \frac{T_0 z}{\varepsilon} = \frac{zQ}{2\varepsilon} \tag{15}$$

Using (7) and (15), we get

$$p(z)\,dz = \frac{2\varepsilon}{Q}\left[\sum_{k=0}^{K} c_k \cos\left(\frac{4\pi\varepsilon n_s}{Q}\,k + \theta_k\right)\right] dn_s \tag{16}$$

Integrating the right-hand side of (16), from n_s to $n_s + 1$ we obtain the probability of creating n_s particles p_{n_s} as

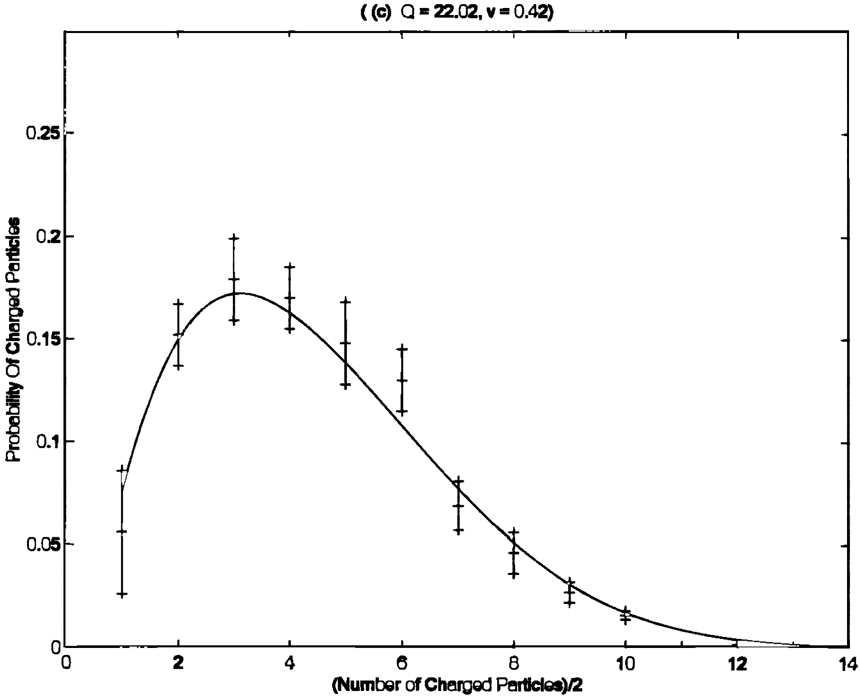


Fig. 2. Continued.

$$p_{n_s} = \frac{2\varepsilon a_0}{Q} + \left[\sum_{k=1}^{K} \frac{c_k}{2\pi k} \sin\left(\frac{4\pi\varepsilon(n_s + 1)}{Q} k + \theta_k \right) - \sum_{k=1}^{K} \frac{c_k}{2\pi k} \sin\left(\frac{4\pi\varepsilon n_s}{Q} k + \theta_k \right) \right] \tag{17}$$

The probability distribution for the emission of a charged pair of pions from one fireball is assumed to be binomial in the form:

$$\Psi(n_s) = \frac{N!}{n_c!\,(N - n_c)!} u^{n_c} v^{(N-n_c)} \tag{18}$$

where N is the total created pairs of particles ($N = n_s/2$), n_c is the number of pairs of charged particles, and u and v are the probabilities that the pair of particles is charged or uncharged (neutral and heavier particles than pions), respectively.

Now, the number of charged particles from one fireball is given by $n = 2n_c + 1$. Then the distribution of charged particles from one fireball is

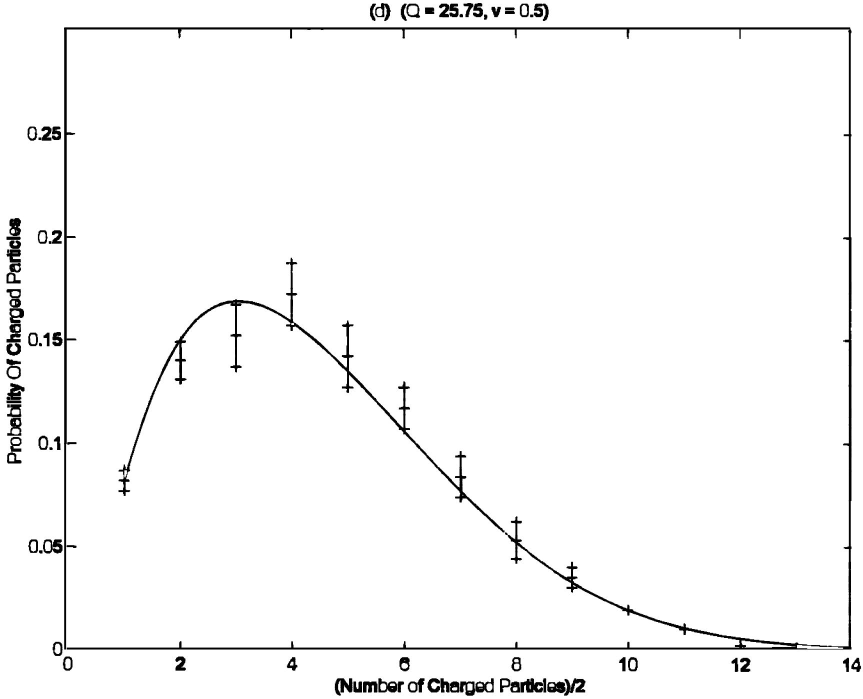


Fig. 2. Continued.

$$\varphi(n) = \sum_{n_s \text{ even}} \psi(n_s) p_{n_s} \tag{19}$$

Because of charge conservation, the probability of getting a number of charged pairs out of an even total number n_s is the same as the probability of getting the same number of charged pairs out of the next odd number, i.e., $n_s + 1$. Therefore, we can calculate the probability of getting any number of charged particles n_c from two fireballs as

$$p_{n_c} = \sum_{n,\text{odd}}^{n_c} \varphi(n)\varphi(n_c - n); \qquad n_c = 2,4, \ldots, Q/\varepsilon \tag{20}$$

Equation (20) has been used to calculate the charged multiplicity distribution for different energies. These distributions are plotted in Fig. 2 along with experimental data [22–25]. The experimental data are shown as vertical lines. The maximum and minimum of these vertical lines represent the maximum positive and negative observations, respectively. The solid line represents the theoretical calculated distribution employing (20). The theoretical distribution has been optimized according to the probability of uncharged particles v in

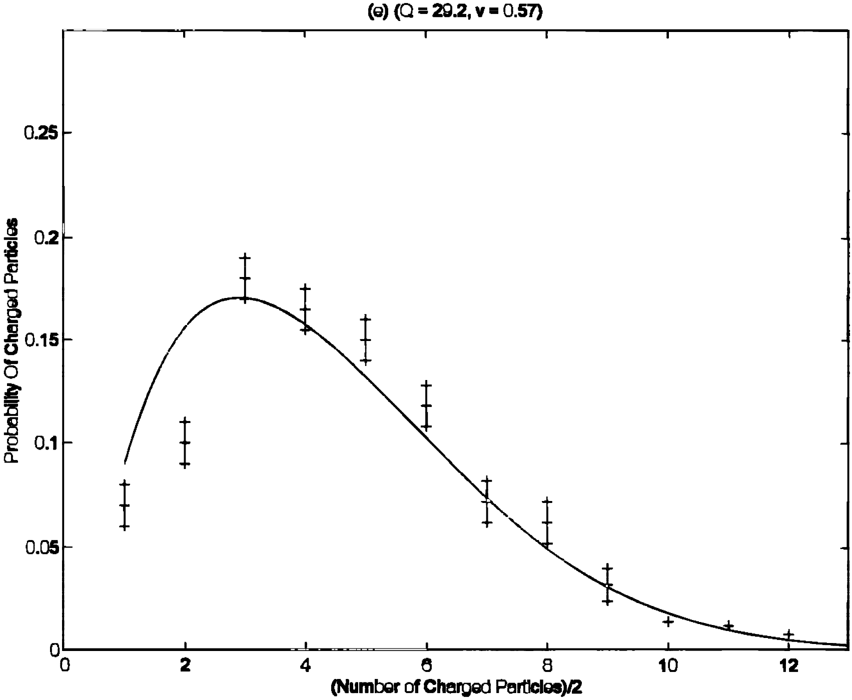


Fig. 2. Continued.

(18). The good agreement between experimental and theoretical distributions is clear. It is observed that the probability of uncharged particles (neutral and heavier particles than pions) increases as the energy level increases. The following illustrates the optimum v for different energies:

v	0.33	0.41	0.42	0.50	0.57	0.60
Q(GeV)	12.03	17.79	22.02	25.75	29.20	43.20

5. CONCLUSION

The proton–proton interaction at high energies has been investigated in the context of a parton two-fireball model. The conventional overlap function (z-function) is shown to be nonlinear and difficult to manipulate mathematically. We proposed a trigonometric Fourier series representation of the z-function. The Fourier series coefficients have been calculated employing trapezoidal numerical integration. It was shown that the resulting trigonometric series facilitates the mathematical analysis. The proposed representation was employed to deduce the p–p interaction cross sections as well as the

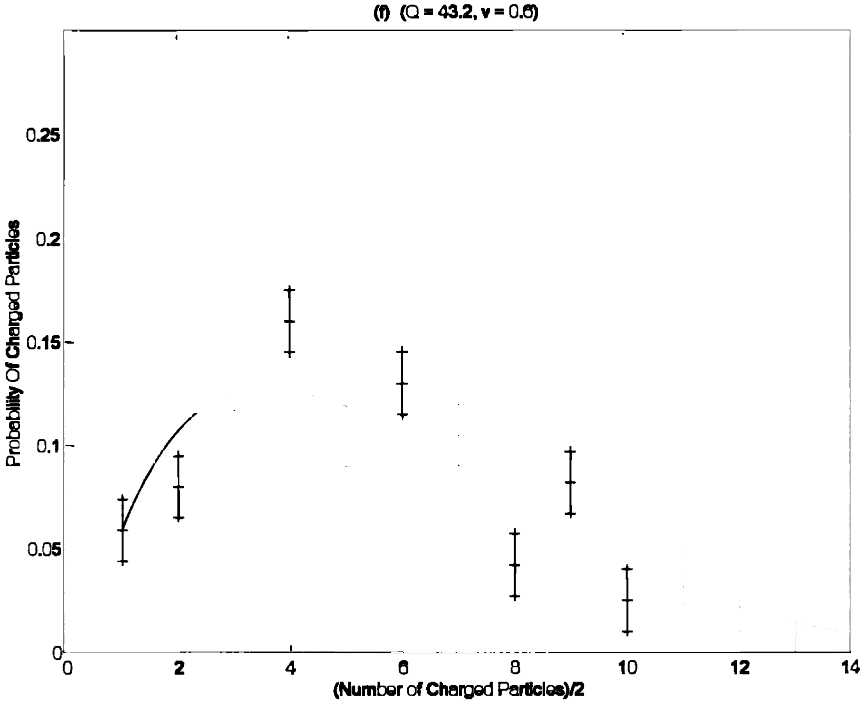


Fig. 2. Continued.

charged multiplicity distribution at a range of energies. The emission of secondary particles is assumed to follow a binomial distribution. Figures comparing the theoretical predictions to experimental data show that the cross-section fitting is somewhat poor at lower energies and good at higher energies. On the other hand, the fitting of the charged-particle distribution is good everywhere. However, this is based on optimizing the probability of the other created particles. It is shown that the probability of neutral and heavier particles increases as the energy increases.

REFERENCES

1. P. Carruthers and C. C. Shih, *Int. J. Mod. Phys. A* **2**, 1547 (1987).
2. Z. Koba, H. B. Nielsen, and P. Olesen, *Nucl. Phys. B* **40**, 317 (1972).
3. A. I. Golokhvastov, *Sov. J. Nucl. Phys.* **27**, 430, (1978); **30**, 128 (1979).
4. A. Giovannini and L. Van Hove, *Z. Phys. C* **30**, 391 (1986); P. K. Mackown and A. W. Wolfendale, *Proc. Phys. Soc. Lond.* **89**, 553 (1966); N. Suzuki, *Prog. Theor. Phys.* **51**, 1625 (1974).
5. G. N. Fowler *et al.*, *Phys. Rev. Lett.* **57**, 2119 (1986).
6. P. Carruthers and C. C. Shih, *Phys. Lett.* **137B**, 425 (1984); *Int. J. Mod. Phys. A* **2**, 1547 (1987).

7 H. M. A. Radi *et al.*, *Phys. Rev. C* **27**, 606 (1983).
8. R. Hagedorn, *Nuovo Cimento Suppl.* **3**, 147 (1965); *Nuovo Cimento Supp.* **6**, 311 (1968); *Nuovo Cimento* **56A**, 1027 (1968); *Nucl. Phys. B* **24**, 93 (1970); J. Ranft, Ref. TH/851-CERN (1967); Ref. TH. 1027-CERN (1969).
9. Cai-Xu, Chao Wei-qin, and Meng Ta-Chung, *Phys. Rev. D* **33**, 1287 (1986).
10. Y. Nambo, The confinement of quarks, *Sci. Am.* **48** (1976).
11. M. Jacob and R. Slansky, *Phys. Rev. D* **5**, 1847 (1972).
12. R. Hwa, *Phys. Rev. D* **1**, 1790 (1970); *Phys. Rev. Lett.* **26**, 1143 (1971).
13. M. Tantawy, M. ElMashad, and M. Y. Elbakry, *Indian J. Phys.* **72A**, 110 (1998).
14. M. S. ElNagdy *et al.*, *Arab J. Nucl. Sci. Appl.* **25**, to appear (1998).
15. M. Tantawy, M. ElMashad, and M. Y. Elbakry, The 3rd ICEMP, Faculty of Engineering, Cairo University, 23–25 December 1997.
16. R. P. Feynman, *Photon–Hadron Interactions*, Benjamin, Reading, Massachusetts (1972).
17. E. Fermi, *Prog. Theor. Phys.* **5**, 570 (1950); *Phys. Rev.* **81**, 683 (1951).
18. J. Ranft, *Phys. Lett.* **31B**, 529 (1970).
19. R. Hagedorn and V. Wambach, Ref. Th.2242 CERN (1976).
20. F. G. Stremler., *Introduction to Communication Systems*, Addison-Wesley, Reading, Massachusetts (1982).
21. V. Flaminio *et al.*, CERN-HERA 79-03 (15 October 1979).
22. C. Bromberg *et al.*, *Phys. Rev. Lett.* **31**, 1563 (1973).
23. L. Voyvodic *et al.*, *Phys. Rev. Lett.* **29**, 516 (1972).
24. F. T. Dao *et al.*, *Phys. Rev. Lett.* **29**, 1627 (1972).
25. UA5 Collaboration, G. J. Alner *et. al.*, *Phys. Lett.* **167b**, 4766 (1986).